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Fractal Vision In Characterizing A Digitized Image

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Abstract

Characterization of a digitized image is often required in nondestructive evaluation. Apart from the existing image processing techniques, a recent method to analyze and quantify complex images has been developed based on the fractal theory. As reported, it is possible to get salient information from any digitized image by evaluating the fractal dimension, the defining characteristic of a fractal. In the present work fractal analysis is performed on digitized images obtained from of domains having inserts. The data, digitized by a digitizer board, is obtained from the image of the domain grabbed by a CCD camera. Analysis is performed to compute the fractal dimension of a digitized image, which represents the quality of the image in a quantitative sense. The results show that the fractal dimension is not only sensitive to the presence of a different region (insert) in an image, but is also sensitive to the relative position of the insert in the domain.

Keywords: Nondestructive Evaluation, Digital Images, Pixels, Fractals, Fractal Dimension.

Introduction

In nondestructive evaluation, it is often necessary to characterize a digitized image in identification of inserts, if any. Fractal theory can be successfully employed in quantitative assessment of digital images. The word *fractal* was first coined by Mandelbrot [1] and according to him, the fractal is a "shape made of parts similar to the whole in some way." The defining parameter of a fractal is the fractal dimension. In respect of a two-dimensional (2-D) digitized surface, the fractal dimension corresponds quite closely to our intuitive notion of roughness. An N by N two-dimensional digitized image contains N^2 pixels, each of which has an individual intensity value attached to it. A 3-D representation of this 2-D image may be conceived by incorporating the third dimension as the intensity of each pixel. Therefore, a 2-D image with different intensity values on its pixels may be thought as an imperfect cube (i.e., a cube with dents) whose fractal dimension should lie between 2 and 3. A digitally rough surface will have a smaller fractal dimension while an image with constant intensity is similar to a cube with no dents and the corresponding fractal dimension will be 3 as per definition.

In the present investigation, fractal theory has been employed to characterize 2-D digitized images in detection of inserts. Following the algorithm proposed by Bhatt et. al. [2], fractal dimensions are computed from the slope of the best line fit (in a least square sense) of the fractal graphs of the images. To demonstrate the utility of the fractal concepts, a 30 by 30 pixel (in all 900 pixels) domain having an insert spread over a region of 8 by 8 pixels is considered. The average intensities of the parent domain are in the range of 32-38 while the same for the pixels representing the insert are in the range of 168-180 in the 0-255 grayscale. The gray intensity values were obtained after taking image of the part of a strip, containing square pills by a CCD camera, and digitizing the image by a suitable board. Once the intensities of the pixels pertaining to the parent domain and the insert are obtained, several other domains with different positions of the insert are simulated.

The fractal analysis shows that it can clearly distinguish between the domains with and without insert. Interestingly, the fractal dimension is not only dependent on the relative size and intensities in the insert region, but it is also sensitive to the relative position of the insert. Different domains are considered in which the insert is shifted along the diagonal and any edge of the domain. Curves depicting variations of the fractal dimensions for different positions of the insert are obtained and they may be used as guidelines in identifying the location of the insert inserts in a domain.

Fractal Theory and Earlier Work

The concept of fractals has been used extensively for graphical simulation of natural phenomena, study of image textures and study of material surfaces. Clouds, mountains, turbulent water, lightning and even music has been shown to have a fractal form. Literature also indicates that fractal theory has been successfully implemented for characterizing statistical images. These statistical images have two important properties such as:

(i) Each segment is statistically similar to all others.
(ii) Segments at different scales are statistically indistinguishable and they often possess a remarkable invariance under changes of magnification.

As mentioned earlier, Mandelbrot developed and popularized a relatively novel class of mathematical functions known as *fractals*, a word coined by Mandelbrot in 1975. Mandelbrot's fractal geometry can model many of the seemingly complex shapes found in nature, which possess striking invariance under changes of magnification. This statistical self-similarity can be characterized by a fractal dimension (which agrees with our intuitive notion of dimension) that need not be an integer. Voss [3] showed the generation of a series of random fractal shapes to provide a visual introduction to the concepts of fractal geometry. One example is the *Von Koch snowflake curve* where the property of self-similarity and dimension is well illustrated. He discussed generation of the curve from a single straight line and subsequently the fractal dimension of a D-dimensional self-similar object was deduced. His work also described the mathematical characterization of Mandelbrot's fractional Brownian motion.

Pentland [4] extended the application shape-from-shading and shape-from-texture methods to real surfaces. He used the model of natural surface shapes to derive a technique for 3-D shape estimation for treating the shaded and textured surfaces in a unified manner. In his book, Falconer [5] provided a general framework for the study of irregular sets by means of Haussdorff measure and Haussdorff dimension, which play a central role in defining the fractal sets and the their corresponding fractal dimension. The notion of random fractal was included.

In recent years, researchers have used fractal theory in order to quantify digital images. Lundahl et al [6] demonstrated the use of fractal theory in analyzing X-ray medical images. Chen et al [7] applied the fractal concept to the classification and analysis of images obtained in X-ray medical imaging. In most of the works, the fractional Brownian motion model has been proved to be appropriate for fractal calculations. Bhatt et al [7] used fractal concept to analyze the quality of the reconstructed images in non-medical areas. They showed that fractal dimension represents quality index of the reconstructed images. The performance of different filter functions, used in the convolution back projection algorithm of tomography was also evaluated using fractal dimensions. Munshi et al [8] investigated the applicability of the fractal concept in the field of nondestructive evaluation of real tomographic images. They applied fractal concept to analyze tomograms obtained by an X-ray CT scanner. Datta et al [9] used fractal theory for analyzing the quality of the ultrasonic C-scan images of glass-epoxy and carbon-epoxy composite laminates containing flaws. The fractal dimensions of different images were used to evaluate the sensitivity of different ultrasonic features to different types of flaws.

Algorithm

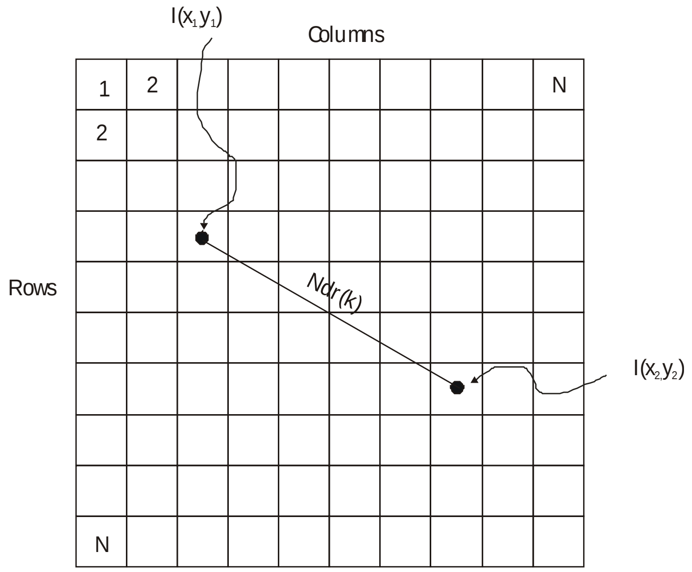

Fig.1: Pixel representation of the digitized domain for fractal analysis

In this section, the algorithm for estimation of the fractal dimension of a 2-D digitized image is discussed. Any NxN 2-D digitized image, as shown in Fig. 1, containing N^2 pixels is considered. Here I(x,y) represents the intensity value of any such pixel with coordinate x,y. The fractal graph of any such image is visualized as the plot of log(NMSID) vs. log(NSR). The NSR corresponds to the normalized scale range vector and it consists of reference scale and generally corresponds to the possible distances between any pair of pixel in the concerned image. Thus

$$NSR = [ndr(1), ndr(2), ..., ndr(k), ..., ndr(m)] \quad (1)$$

where

$$ndr(k) \le \left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]^{\frac{1}{2}} < ndr(k+1) \quad (2)$$

The NMSID vector corresponds to the Normalized Multi-Scale Intensity Difference vector. It consists of different absolute-intensity difference averages around each normalized reference scale (NSR), i.e.,

$$NMSID = [ndi(1), ndi(2), ..., ndi(k), ...ndi(m)] \quad (3)$$

where

$$ndi(k) = \frac{\sum_{x_1=1}^{N}\sum_{y_1=1}^{N}\sum_{x_2=1}^{N}\sum_{y_2=1}^{N}\left|I(x_2, y_2) - I(x_1, y_1)\right|}{npn(k)} \quad (4)$$

and NPN is the Normalized Pixel pair Number vector, which consists of elements that represent the number of pixel pairs with scale (distance) values similar to the reference scale. Plotting log(NMSID) vs. log(NSR) for I=1,2, …, M, results in a curve consisting of M points which is the fractal graph of the corresponding image. A linear fractal graph represents a perfect fractal, otherwise a least square linear regression on it gives the slope H of the resultant curve. The fractal dimension FD is then calculated from the relation

$$FD = 3 - H \quad (5)$$

The significance of the constant term 3 in the above equation is already discussed in the beginning. Any digitized image with different intensity values on its pixels is conceived as an imperfect cube in which the fractal dimension should be lie in between 2 and 3. Moreover, it is important to note that the FD, evaluated as above, will vary since the slope of the best linear fit depends on the range-of-scales of distances being selected. It is a common practice to consider the range of NSR in which the fractal graph exhibits linear behaviour.

Present Work

The present work is to discuss a case study regarding evaluation of digitized images using fractal theory stated above. The concerned image, used in this work is that of a pill in a strip, taken by means of a CCD camera and digitized by a digitizer board. The present work may be used in automating the inspection procedure along a pharmaceutical line in which one might want to know if a blister package is properly filled with the correct pills and the correct quantity of pills. There may be other methods in doing this, the present method is aimed to achieve this goal in an automated manner using the fractal theory.

An image domain consisting of 30x30 pixel is considered in which the pill area is represented by a 8x8 pixel group. The objective is to ascertain whether the existence of the pill, here acting as an insert, can be detected by fractal analysis. Once that is done, analysis is conducted on other simulated domains using the same digitized data but with different locations of the insert. The objective is to get an idea about the sensitivity of fractal dimension with the relative location of any insert in a domain. Before presenting the results, the acquisition procedure of digitized data is discussed briefly in the following section.

Data Acquisition

The digitized data acquisition procedure of a section of a strip containing pills is done on a conveyer, CCD camera and a computer (installed with an appropriate digitizer board) assembly. The strip is placed on the properly illuminated conveyor, supposed to be coming out from the packaging section. The CCD camera, used for image grabbing, is connected to the digitizer Board, which in turn, is controlled by the corresponding software. It may be noted that in order for computer to process images, the images must be numerically represented. This process is known as image digitization. The digitization process divides an image into a two dimensional grid of small units called pixels. Each pixel has a value that corresponds to the intensity or colour at that location in the image.

To enable image grabbing, specification of a board capable of acquisition and a camera upon installation of the software is necessary. Grabbing is initiated in continuous mode into the window specified with the options preferences command. The menu driven data acquisition software has the following grab control options namely *freeze, snapshot, continuous, close* and *adjust*. Continuous mode is very useful for adjusting and focusing the camera. Once the image has been frozen, one can continue grabbing one snapshot at a time, or restart in continuous mode. While grabbing, the software also allows to adjust the black and white reference levels of the frame grabber's analog-to-digital converters or the hue and saturation of its composite decoder. In case of grabbing from a grayscale camera, the Video Signal References allows one to change either the brightness and contrast of the grabbed image or its black and white reference levels. In case of grabbing from an RGB camera, the Video Signal References allow to adjust the black and white reference levels of each component.

Results and Discussion

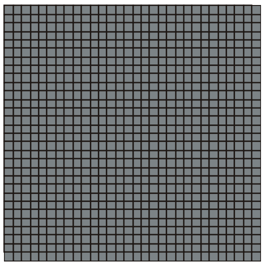

Fig. 2a: Digitized domain without Insert

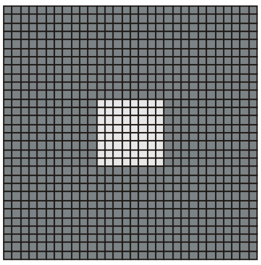

Fig. 2b: Digitized domain with Insert

Following the algorithm mentioned in a preceding paragraph, a computer program is generated for fractal analysis of a digitized image. The program takes the coordinate and corresponding intensity values of the pixels in the concerned digitized image as input and outputs the fractal dimension of the image for a user chosen scale in the spatial coordinate. In order to check the utility of the program, two digitized domains, as shown in Figure 2a and Figure 2b, are considered. Figure 2a represents the domain without any insert while Figure 2b depicts the domain with insert located centrally. Both the domains consist of 30x30 pixel array while the insert region is spread over an 8x8 grid of pixels. The average intensity of the base domain is in the range of 32-38 while the insert intensities vary from 168-180 in the 0-255 grayscale.

The fractal graphs, as a plot of log(NSR) vs. log(NMSID), for both the cases, are shown in Fig. 3. The fractal graph for the domain with the insert is distinctly different from that of the domain without any insert. Linear regression, in a least square sense, is performed on the data and the slopes of the best fit lines are found to be 0.01532 (FD=2.98468) and 0.975 (FD=2.025) for the domain without insert and domain with insert respectively. The result clearly shows that the fractal analysis is capable of detecting the presence of an insert in a domain.

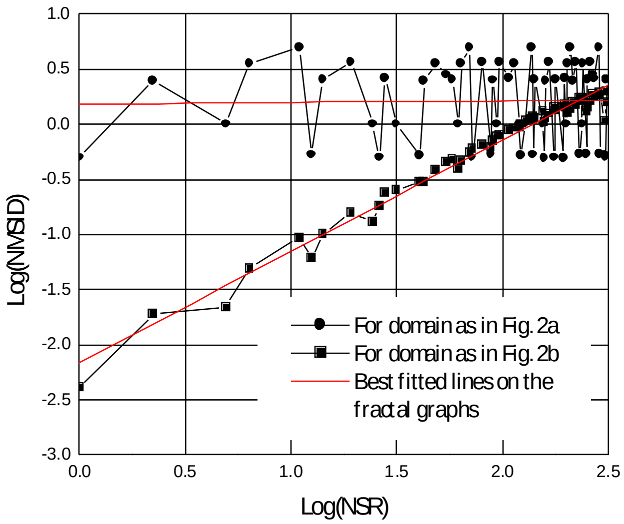

Fig. 3: Fractal Graphs for the domains with insert and without insert

In order to investigate the sensitivity of fractal dimension to the relative position of the insert in a sample, several domains are simulated using the same data for the pixels constituting the base material and the insert. In this investigation, the location of the insert is changed along the diagonal of the domain, i.e., in all 12 different domains are analyzed. Typical domains having inserts located at a corner, at an intermediate position and at the central position are shown in Figures 4a, 4b and 4c respectively. Fractal dimensions are calculated for all the cases and their variations are shown in Figure 5. The horizontal axis of the plot corresponds to the position of the square insert in the base material (also square). Position 1 corresponds to the case where the top left corner of the insert coincides with the top left corner of the base material. Position 2 corresponds to the domain where the top left corner of the insert occupies the next position along the diagonal towards the centre of the base domain and so on. The variations show that sensitivity of the detection increases with the insert approaching the center of the base material. Increased sensitivity means that the fractal dimension is considerably less and close to 2 compared to the case of the domain having no insert. Two important conclusions can be drawn from the plot; (i) there is not much change in the FD value in the last five cases in which the insert is close the center, (ii) The first case (i.e., position 1) is an exception to the above observation. Here the fractal dimension is less than the next case and hence is more sensitive to the insert in this position.

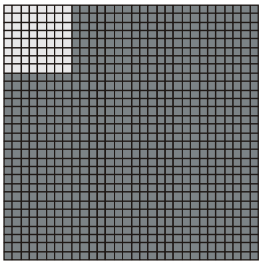

Fig. 4a: Position 1

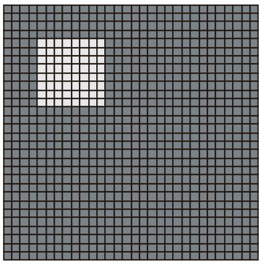

Fig. 4b: Position 5

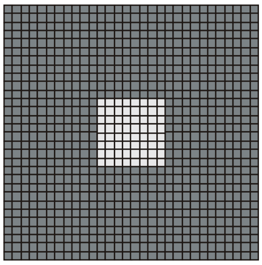

Fig. 4c: Position 12

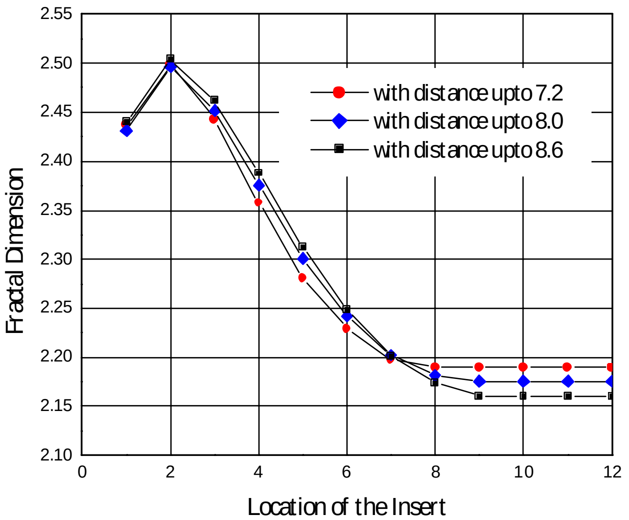

Fig. 5: Variation of the FD of the domain with insert at different locations

It may be noted that the FD variation is also sensitive to the scale of range being used to obtain the best fit line of the fractal graph in a least square sense. In all three scales are used and shown, namely, (i) 7.211, (ii) 8.062, and (iii) 8.602 which are all close to 8, the insert size. In case of an insert of unknown size, various trials may be undertaken with different scales and changes in values may be noticed.

Conclusion

In the light of the discussions above, following conclusions may be drawn. The fractal theory is capable to differentiate between domains with or without insert. It is also possible to get a first hand information about the possible location of an insert, if any, in the domain under consideration. The variation pattern of the FD with different positions of the insert may act as a guideline for automated detection of them. A potential application of this may be found in checking the filling of pills in a blister package in a pharmaceutical industry.

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